GRAPHS OF CUBIC FUNCTIONS (LIVE) 18 MAY 2015

Section A: Summary Notes

The standard form of a cubic graph is:

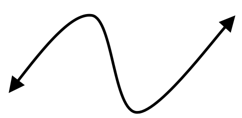
$$y = ax^3 + bx^2 + cx + d$$

When plotting a graph, there are 4 steps to follow:

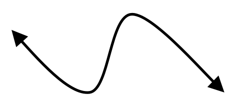
- Determine the x-intercept LET $Y = 0$
- Determine the y-intercept LET $X = 0$
- Determine the STATIONARY POINTS (also known as the turning points) this is done by letting $f'(x) = 0$
- Substitute the x values of the stationery points into the original equation to calculate the corresponding y values.

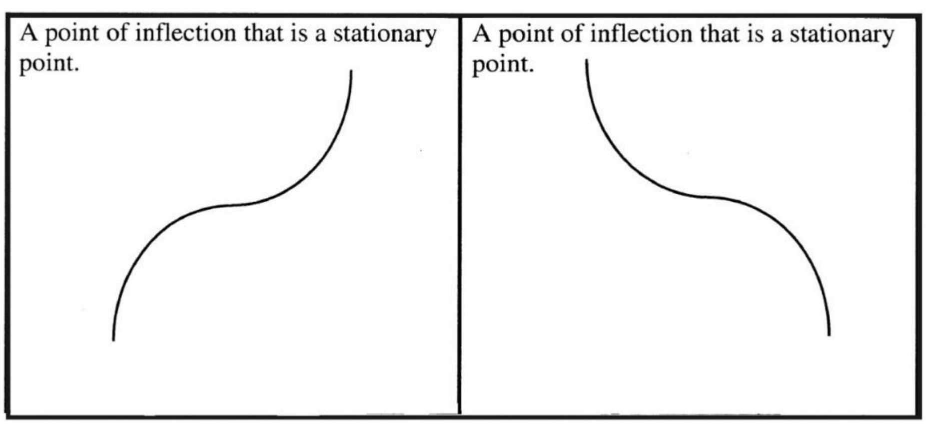
Remember:

$a \rangle 0$



$a \langle 0$





Section B: Exam practice questions

Question 1

Sketch the graph of

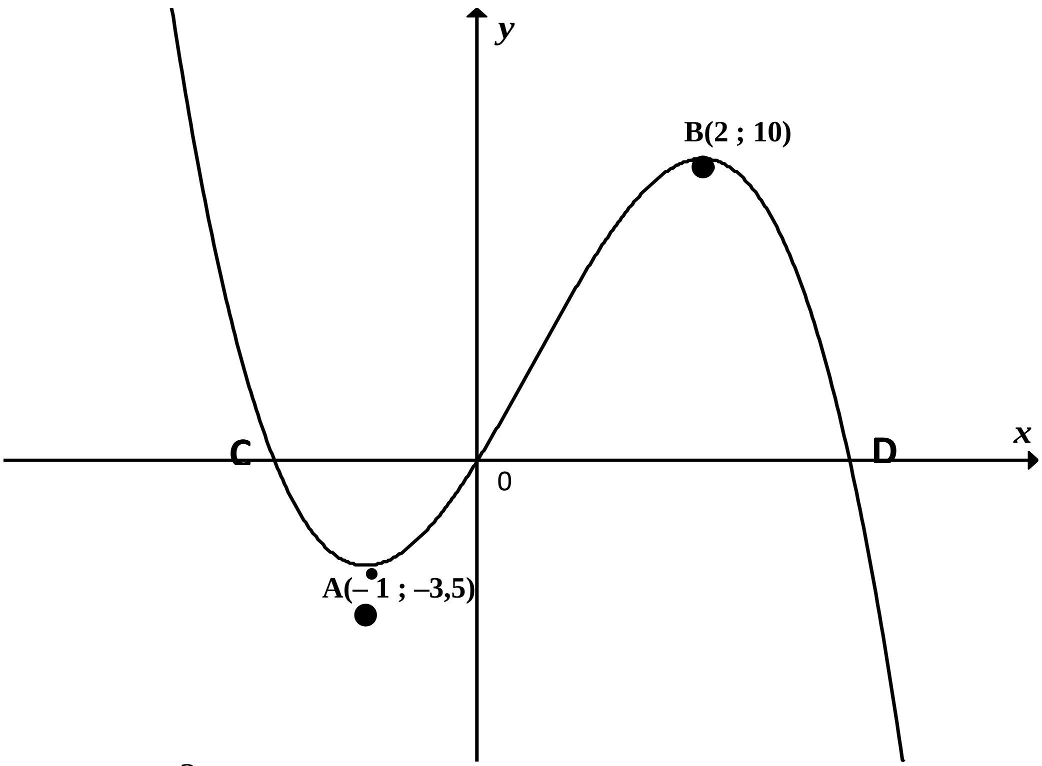
a) $f\left(x\right) = x^3 - 9x^2 + 24x - 20$

b) $f(x) = x^3 + 2x^2 - 4x - 8$

Showing all intercepts with the axes and any stationary points.

Question 2

The graph of $h(x) = -x^3 + ax^2 + bx$ is shown below. $\mathrm{A}(-1\,;-3{,}5)$ and $\mathrm{B}(2\,;10)$ are the turning points of *h*. The graph passes through the origin and further cuts the *x*-axis at C and D.



2.1 Show that $a = \frac{3}{2}$ and $b = 6$ (7)

2.2 Calculate the average gradient between A and B. (2)

2.3 Determine the equation of the tangent to *h* at $x = -2$. (5)

2.4 Determine the *x*-value of the point of inflection of *h*. (3)

Question 3

Sketch the graph of $f(x) = 2x^3 - 6x - 4$ (17)

Question 4

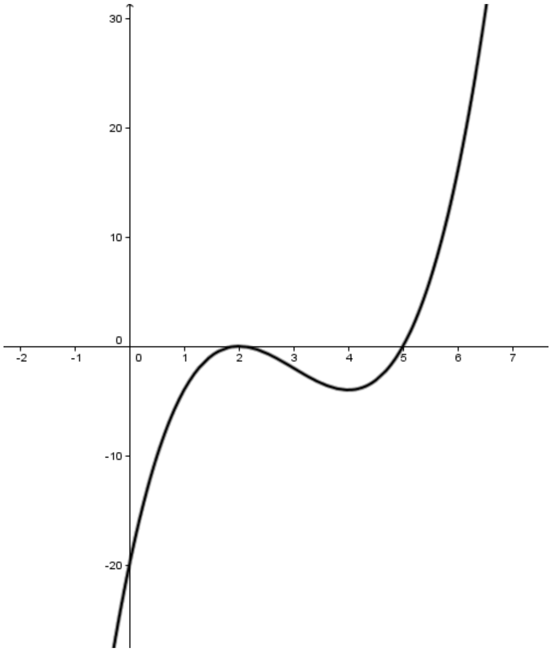
Sketch the graph of $f(x) = x^3 - 3x^2 + 4$

Indicate the coordinates of the stationary points, intercepts with the axes and any points of inflection. (15)

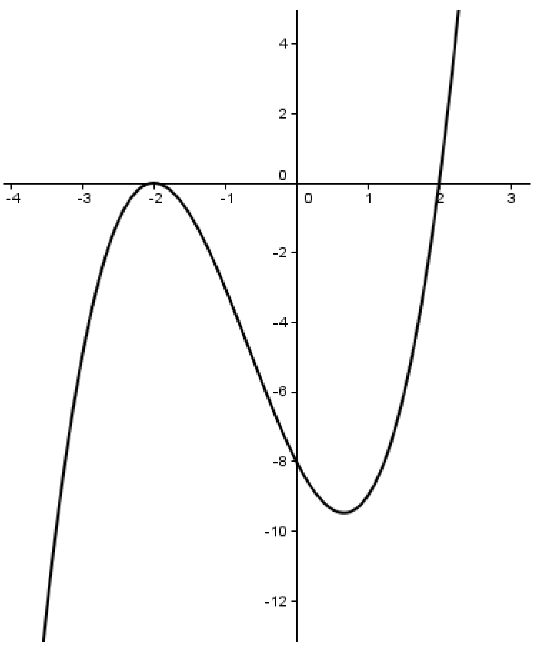
Section B: Exam practice questions

Question 1

a)



b.)



Question 2

2.1	$h'(x) = -3x^2 + 2ax + b$ $h'(-1) = -3(-1)^2 + 2a(-1) + b$ $0 = -3 - 2a + b$ $2a - b = -3 \quad \ldots \text{(i)}$ $h'(2) = -3(2)^2 + 2a(2) + b$ $0 = -12 + 4a + b$ $4a + b = 12 \quad \ldots \text{(ii)}$ $6a = 9 \qquad \text{(i)} + \text{(ii)}$ $\therefore a = \frac{3}{2}$ $\therefore 2\left(\frac{3}{2}\right) - b = -3$ $b = 6$	✓ $h'(x) = -3x^2 + 2ax + b$ ✓ $h'(-1) = -3(-1)^2 + 2a(-1) + b$ ✓ $2a - b = -3$ ✓ $h'(2) = -3(2)^2 + 2a(2) + b$ ✓ $4a + b = 12$ ✓ $a = \frac{3}{2}$ ✓ $b = 6$ (7)
2.2	Average gradient $= \frac{10 - (-3,5)}{2 - (-1)}$ $= \frac{13,5}{3}$ $= \frac{9}{2}$	✓ $\frac{10 - (-3,5)}{2 - (-1)}$ ✓ $\frac{9}{2}$ (2)
2.3	$h(x) = -x^3 + \frac{3}{2}x^2 + 6x$ $\therefore h'(x) = -3x^2 + 3x + 6$ $h'(-2) = -3(-2)^2 + 3(-2) + 6$ $h'(-2) = -12$ Point of contact $(-2\,;\,2)$ $y - 2 = -12(x + 2)$ $y = -12x - 22$	✓ $h(x) = -x^3 + \frac{3}{2}x^2 + 6x$ ✓ $h'(x) = -3x^2 + 3x + 6$ ✓ $h'(-2) = -12$ ✓ $y = -12x - 22$ ✓ $h'(-2) = -12$ (5)

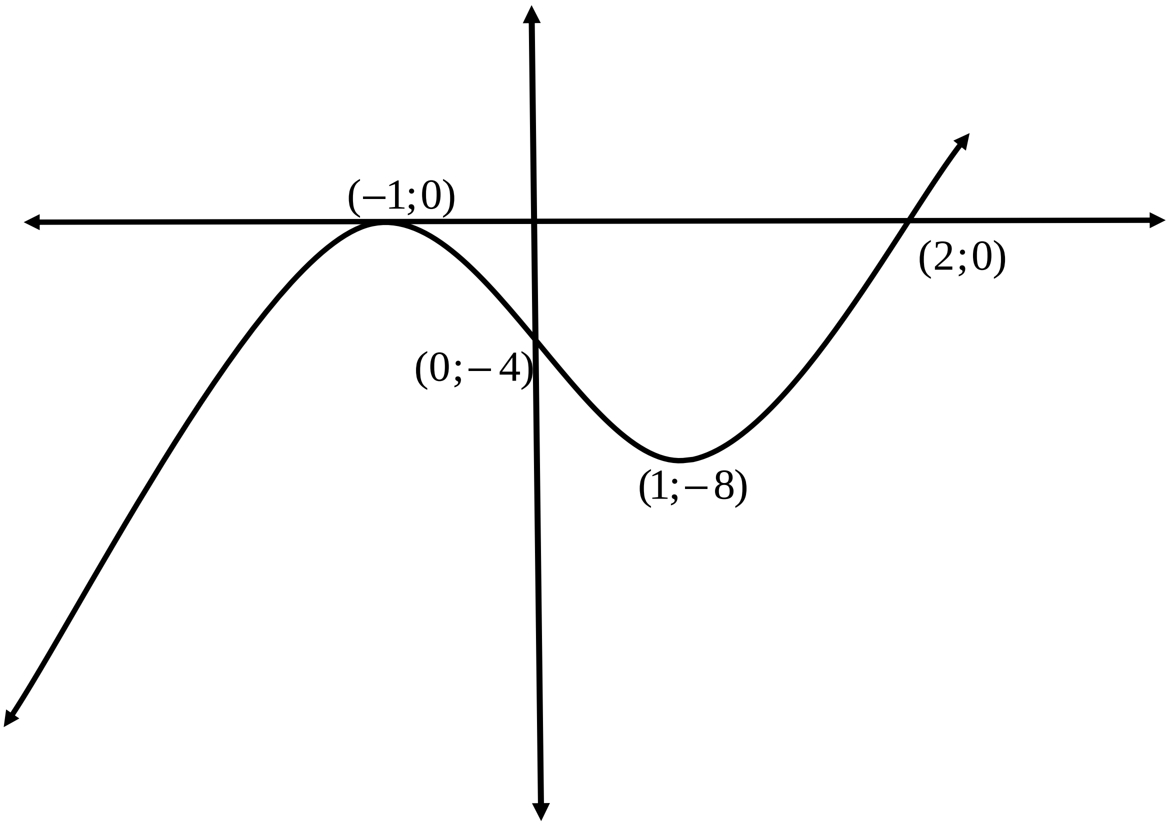
2.4	$h'(x) = -3x^2 + 3x + 6$ $h''(x) = -6x + 3$ $-6x + 3 = 0$ $x = \frac{1}{2}$	✓ $h''(x) = -6x + 3$ ✓ $-6x + 3 = 0$ ✓ $x = \frac{1}{2}$ (3)

[17]

Question 3

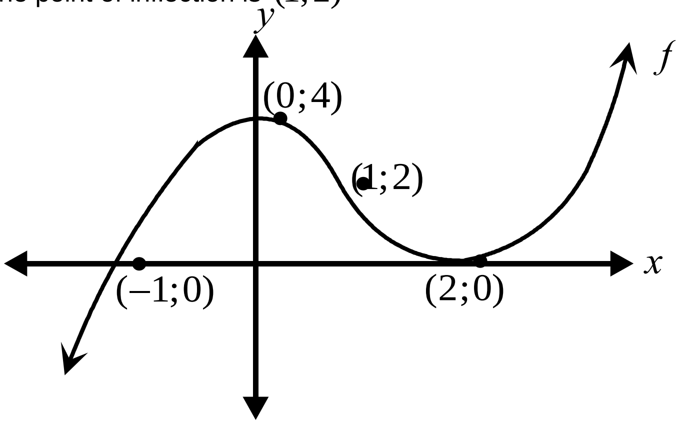
	***y*-intercept:** $(0;-4)$ ***x*-intercepts:** $0 = 2x^3 - 6x - 4$ $\therefore 0 = x^3 - 3x - 2$ $\therefore 0 = (x+1)(x^2 - x - 2)$ (using the factor theorem) $\therefore 0 = (x+1)(x-2)(x+1)$ $\therefore x = -1$ or $x = 2$ $(-1;0)$ $(2;0)$ **Stationary points:** $f(x) = 2x^3 - 6x - 4$ $\therefore f'(x) = 6x^2 - 6$ $\therefore 0 = 6x^2 - 6$ (At a turning point, $f'(x) = 0$) $\therefore 0 = x^2 - 1$ $\therefore x = \pm 1$ $f(1) = -8$ $f(-1) = 0$ Turning points are $(1;-8)$ and $(-1;0)$ **Point of inflection:** $f'(x) = 6x^2 - 6$ $\therefore f''(x) = 12x$ $\therefore 0 = 12x$ $\therefore x = 0$ $f(0) = -4$ Point of inflection at $(0;-4)$ Alternatively: The *x*-coordinate of the point of inflection can be determined by	✓ $(0;-4)$ ✓ $0 = 2x^3 - 6x - 4$ ✓ $0 = (x+1)(x^2 - x - 2)$ ✓ $0 = (x+1)(x-2)(x+1)$ ✓ $(-1;0)$ $(2;0)$ ✓ $f'(x) = 6x^2 - 6$ ✓ $0 = 6x^2 - 6$ ✓ $x = \pm 1$ ✓ $(1;-8)$ and $(-1;0)$ ✓ $f''(x) = 12x$ ✓ $(0;-4)$ ✓ $\frac{(1)+(-1)}{2}$

	adding the x-coordinates of the turning points and then dividing the result by 2. $x=\frac{(1)+(-1)}{2}=0$	✓ $x=0$



Question 4

	x-intercepts: *y*-intercept: 4 $0=x^3-3x^2+4$ $\therefore (x+1)(x^2-4x+4)=0$ $\therefore (x+1)(x-2)(x-2)=0$ $\therefore x=-1$ or $x=2$ $f'(x)=3x^2-6x$ $\therefore 0=3x^2-6x$ $\therefore 0=x^2-2x$ $\therefore 0=x(x-2)$ $\therefore x=0$ or $x=2$ For $x=0$ $f(0)=(0)^3-3(0)^2+4=4$	✓ $0=x^3-3x^2+4$ ✓ $(x+1)(x^2-4x+4)=0$ ✓ $x=-1$ or $x=2$ ✓ $f'(x)=3x^2-6x$ ✓ $0=3x^2-6x$ ✓ $x=0$ or $x=2$ ✓ $(0;4)$

	Max turning point at $(0\,;4)$ For $x=2$ $\quad f(2)=(2)^3-3(2)^2+4=0$ Min turning point at $(2\,;0)$ $f'(x)=3x^2-6x$ $\therefore\ f''(x)=6x-6$ $\therefore 0=6x-6$ $\therefore -6x=-6$ $\therefore x=1$ $f(1)=(1)^3-3(1)^2+4$ $f(1)=2$ The point of inflection is $(1;2)$ 	✓ $(2\,;0)$ ✓ $f''(x)=6x-6$ ✓ $x=1$ ✓ $(1;2)$ ✓ intercepts with the axes ✓ turning points ✓ shape ✓ point of inflection **[15]**